

On the invariance of the speed of light

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Abstract

The invariance of the speed of light in all inertial frames - the second postulate of special theory of relativity (STR) - is shown to be an inevitable consequence of the relativity principle of special theory of relativity taken in conjunction with the homogeneity of space and time in all inertial frames, i.e., the 1st postulate of STR. The new approach presented here renders the learning of special theory of relativity logically simpler, as it makes use of only one postulate.

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1 Introduction

The two postulates of special relativity, enunciated by Einstein[1], viz.(i) the relativity principle: the laws by which the states of physical systems undergo changes are not affected, whether these changes be referred to the one or the other of two systems of coordinates in uniform translatory motion; and (ii) the postulate on the constancy of the speed of light: any ray of light moves in the “stationary” system of coordinates with the determined speed c , whether the ray be emitted by a stationary or moving body; are the basis of a theory – the special theory of relativity – that has been tremendously successful in describing a wide range of phenomena, although there exist derivations of Lorentz transformations (LT) (that establishes a kinematical connection between space and time) without the postulate on the speed of light [2, 3, 4, 5, 6, 7, 8, 9]. However, in this communication, the invariance of the speed of light in all inertial frames [postulate (ii) above] is shown to be a natural consequence of the relativity principle [postulate (i) above] taken in conjunction with the homogeneity of space and time in all inertial frames. The approach presented here is logically simple and new.

2 Invariance of the speed of light as consequence of the relativity principle

Consider two inertial frames F and F' which are in uniform relative motion along a common X and X' axis with corresponding planes parallel and let the velocity of F' with respect to F be \mathbf{v} along the positive X -axis of F . An event may be characterized by specifying the coordinates (x, y, z, t) of the event in F and the same event is characterized by the coordinates (x', y', z', t') in F' . Let us proceed to find a transformation between (x, y, z, t) and (x', y', z', t') and then deduce from it Einstein’s postulate on the velocity of light in special relativity. Suppose that at the instant the origins 0 and $0'$ coincide, we let the clocks there to read $t = 0$ and $t' = 0$ respectively. The homogeneity of space and time in inertial frames requires that the transformations must be linear (see for example [10]), so that the simplest form they can take is

$$x' = k(x - vt); \quad y' = y; \quad z' = z; \quad t' = lx + mt \quad (1)$$

In order to determine the values of the three co-efficients k, l , and m , let us imagine that at instant when the origins of F and F' coincide (i.e., at $t = t' = 0$), a light wave left a point source kept stationary at the origin of F . Since the space is homogeneous and isotropic in F , the light originating from the stationary point source in F would spread uniformly in all directions with the determined speed c (say). As the source is moving with velocity $(-\mathbf{v})$ with respect to F' , let us assume that the speed of light in F' as c' , assuming we are unaware of Einstein’s postulate on the constancy of the speed of light in different inertial frames. Some time after the moment $t = t' = 0$, when the clocks of F read a time t , suppose that the clocks of F' read a time t' . The wavefront of the light wave looks spherical in F , because the wave originating from a stationary point source moves in the homogeneous and isotropic space of F . But we are yet uncertain

about the shape of the same wavefront in F' , because we know not yet whether c' is uniform in all directions of F' or not. Now let us consider the space and time coordinates of an arbitrary point on the same wavefront observed in F as (x, y, z, t) and the space and time coordinates of the same point on the same wavefront observed in F' as (x', y', z', t') . So the space coordinates (x, y, z) of the wavefront in F differ from point to point of the wavefront and similarly the space coordinates (x', y', z', t') of the wavefront in F' also differ from point to point. But the time coordinate t is the same for all points of the wavefront in F and similarly the time coordinate t' is the same for all points of the wavefront in F' [1]. Then for an arbitrarily chosen point on the wavefront characterized by the space-time coordinates (x, y, z, t) and (x', y', z', t') respectively, we must have the following relations for the same point on the same wavefront as observed in F and F' :

$$x^2 + y^2 + z^2 = c^2 t^2 \quad (2)$$

$$x'^2 + y'^2 + z'^2 = c'^2 t'^2 \quad (3)$$

Substituting the transformations (1) in (3), one gets:

$$(k^2 - l^2 c'^2) x^2 + y^2 + z^2 - 2(l m c'^2 + k^2 v) x t = (m^2 c'^2 - k^2 v^2) t^2 \quad (4)$$

For Eq.(4) to agree with Eq.(2), we must have

$$\begin{aligned} (i) \quad & k^2 - l^2 c'^2 = 1, \quad \text{or} \quad l^2 = (k^2 - 1)/c'^2 \\ (ii) \quad & m^2 c'^2 - k^2 v^2 = c^2, \quad \text{or} \quad m^2 = (c^2 + k^2 v^2)/c'^2 \\ (iii) \quad & l m c'^2 + k^2 v = 0, \quad \text{or} \quad l m = -k^2 v/c'^2 \end{aligned} \quad (5)$$

Equations (5) when solved for k, l and m yield

$$\begin{aligned} (i) \quad & k = (1 - v^2/c^2)^{-1/2} \\ (ii) \quad & l = -(v/cc')k \\ (iii) \quad & m = (c/c')k \end{aligned} \quad (6)$$

With these values of k, l and m , the transformations (1) can be represented in the following matrix form, viz.,

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = A \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}, \quad \text{where } A = \begin{pmatrix} k & 0 & 0 & -kv \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -kv/cc' & 0 & 0 & kc/c' \end{pmatrix} \quad (7)$$

The inverse transformations for x, y, z, t , are then given by

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = A^{-1} \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix}, \quad \text{where } A^{-1} = \begin{pmatrix} k & 0 & 0 & kv c'/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ kv/c^2 & 0 & 0 & kc'/c \end{pmatrix} \quad (8)$$

Hence the transformation for x becomes

$$x = k(x' + vc't'/c) \quad (9)$$

But relativity principle demands that the transformation for x must be given by [10]:

$$x = k(x' + vt') \quad (10)$$

Hence for Eq.(9) to be in accord with the relativity principle (so with Eq.(10)), we must have $c = c'$.

3 Conclusion

We have, thus, shown that the constancy of the speed of light in all inertial frames is an inevitable consequence of the relativity principle taken in conjunction with the homogeneity of space and time in all inertial frames. The present approach renders the theory of special relativity logically simpler as it makes use of only one postulate. The idea of constructing a relativity theory by using only the relativity principle has also been discussed by Ritz, Tolman and Pauli [11], but the present approach here is new and simpler and has been given as preliminary report in [12].

References

- [1] A. Einstein, “On the Electrodynamics of Moving Bodies”, in *The Principle of Relativity: A collection of original Memoirs on the Special and General Theory of Relativity*, (Dover Publications, Inc., New York, 1952), pp.41.
- [2] W.V. Ignatowsky, *Phys. Zeits.* **11**, 972 (1910).
- [3] L.A. Pars, *Phil. Mag.* **42**, 249 (1921).
- [4] W. Rindler, *Essential Relativity, 2nd Ed.* (Springer-Verlag, New York, 1977), p.51.
- [5] V. Berzi and V. Gorini, “Reciprocity principle and Lorentz transformations”, *J. Math. Phys.* **10**, 1518 (1969).
- [6] A. R. Lee and T. M. Kalotas, “Loerntz transformation from the first postulate”, *Am. J. Phys.* **43**, 434 (1975).
- [7] J. M. Lévy-Leblond, “One more derivation of the Lorentz transformation,” *Am. J. Phys.* **44**, 271 (1976).
- [8] J. H. Field, “Space-time exchange invariance: Special relativity as a symmetry principle”, *Am. J.Phys.* **69(5)**, 569 (2001).

- [9] O. L. Lange, Comment on “Space-time exchange invariance: Special relativity as a symmetry principle”, by J. H. Field [Am. J. Phys. 69(5), 569-575 (2001)], Am. J. Phys. **70**, 78 (2002).
- [10] A.N. Matveev, *Mechanics and Theory of Relativity, Eng. Translation* (Mir. Publishers, Moscow, 1989), pp.101.
- [11] W. Ritz, Ann. Chim. Phys. **13**, 145 (1908); R.C. Tolman, Phys. Rev. **30**, 291 (1910); W. Pauli, *Theory of Relativity* (Pergamon, Oxford, 1956), pp.5-9.
- [12] H. Behera, *Bulletin of Orissa Physical Society*, Vol. X, 153 (2002).